Ninth exercise sheet for the lecture

Syntax Directed Semantics (Part II)

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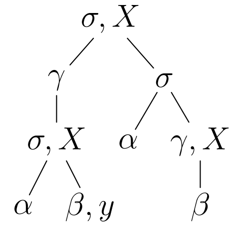
Exercise 25
Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$. Give a $\varphi \in \mathit{MSO}(\Sigma)$ such that $L(\varphi)$ is precisely the set of all $t \in T_\Sigma$ with an even number of σ-symbols. $\diamond$

Exercise 26
Let Δ be a ranked alphabet, $\Gamma_1, \Gamma_2 \subseteq \Delta$, $i \geq 1$. Show $L_{\Gamma_1,\Gamma_2,i} \in \mathit{Rec}(\Delta)$. $\diamond$

Exercise 27
Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and $\varphi = \exists x.\mathit{label}_\gamma(x)$. Use the construction from the lecture to show that $L(\varphi) \in \mathit{Rec}(\Sigma)$. $\diamond$

Exercise 28
Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$, $\mathcal{V} = \{y, X\}$, and let $s \in T^{v}_{\Sigma_{\mathcal{V}}}$ be as follows:



Give $(\llbracket\varphi\rrbracket_{\mathcal{V}}, s)$ for the following choices of $\varphi \in \mathit{MSO}(\mathbb{N}, \Sigma)$:

1. $\varphi = \forall x.\ \neg \mathit{label}_\beta(x) \vee 2$
2. $\varphi = \forall x.\ \neg \mathit{label}_\beta(x) \vee (\mathit{label}_\beta(x) \wedge 2)$
3. $\varphi = \exists x.\ x \in X \wedge (\mathit{edge}_2(x, y) \vee 2)$
4. $\varphi = \forall x.\forall y.\ 2$
5. $\varphi = \exists Y.\forall x.\ \neg(x \in Y) \vee (x \in Y \wedge x \in X)$ $\diamond$

Exercise 29

Let $\underline{K} = (K, \oplus, \odot, 0, 1)$ be a finite, commutative semiring and let $\Sigma = \{\oplus^{(2)}, \odot^{(2)}\} \cup \{k^{(0)} \mid k \in K\}$. Try to give a closed formula φ such that for every $t \in T_\Sigma$, $(\llbracket \varphi \rrbracket, t)$ is the "evaluation" of t in $\underline{K}$. $\diamond$