Eighth exercise sheet for the lecture

Syntax Directed Semantics (Part II)

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Exercise 22
Let $\Sigma = \{\alpha\}$ and $S = (\mathbb{N}, +, \cdot, 0, 1)$. Consider the uwta $M = (Q, \delta, \gamma)$ over Σ and S with $Q = \{q_1, q_2\}$, $\gamma(q_1) = 1$, $\gamma(q_2) = 0$, $\delta(q_1, \alpha) = [\![M']\!]$, and $\delta(q_2, \alpha) = \tilde{3}$, where $\tilde{3} \in S^{rec}\langle\langle Q^*\rangle\rangle$ is the constant series mapping to 3 and $M' = (P, \lambda, \mu, \nu)$ with $P = \{p_1, p_2\}$, $\lambda(p_1) = 2$, $\lambda(p_2) = 0$, $\nu(p_1) = \nu(p_2) = 1$, and μ given as follows:

$\mu(q_1)$	p_1	p_2
p_1	0	2
p_2	1	0

$\mu(q_2)$	p_1	p_2
p_1	0	3
p_2	1	0

1. Compute $([\![M]\!], \alpha(\alpha, \alpha, \alpha))$.
2. Use the construction from the lecture to give a noncircular wmd query equivalent to M. ◇

Exercise 23
Consider the construction from the lecture transforming a uwta into an equivalent noncircular wmd query.

1. Why does it depend on the commutativity of the underlying semiring?
2. Do you think the construction can be "fixed" to work also for arbitrary semirings?
3. If not, conjecture a uwta lacking an equivalent wmd query. ◇

Exercise 24
Define a macro $path(x, y)$ describing that x is an "ancestor node" of y. That is, give an MSO formula φ with free variables x and y such that for every ranked alphabet Σ, $\zeta \in T_\Sigma$, and $\rho : \{x, y\} \to pos(\zeta)$:

$$(\zeta, \rho) \models \varphi \;\text{ iff }\; \exists w \in \mathbb{N}^*.\ \rho(x)w = \rho(y). \qquad ◇$$