Third exercise sheet for the lecture

Syntax Directed Semantics (Part II)

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Exercise 7 (claim from lecture)
Let S be a field, Σ be a ranked alphabet, and $r \in S\langle\langle T_\Sigma \rangle\rangle$. Let $k, n \geq 0$, $\sigma \in \Sigma^{(k)}$, $\xi_1, \ldots, \xi_k, \eta_1, \ldots, \eta_n \in T_\Sigma$, and $a_{1,1}, \ldots, a_{k,n} \in S$ be such that for every $1 \leq i \leq k$,

$$\xi_i^{-1} r = a_{i,1}(\eta_1^{-1} r) + \cdots + a_{i,n}(\eta_n^{-1} r)$$

Show that then for every $1 \leq l \leq k$ and $1 \leq j_1, \ldots, j_{l-1} \leq n$,

$$\sum_{1 \leq j_l \leq n} a_{l,j_l}(\sigma(\eta_{j_1}, \ldots, \eta_{j_{l-1}}, \eta_{j_l}, \xi_{l+1}, \ldots, \xi_k)^{-1} r)$$
$$=$$
$$\sigma(\eta_{j_1}, \ldots, \eta_{j_{l-1}}, \xi_l, \xi_{l+1}, \ldots, \xi_k)^{-1} r \qquad \diamond$$

Exercise 8 (algebraic characterization)
Let $\Sigma = \{\sigma^{(2)}, \delta^{(1)}, \alpha^{(0)}\}$ and let $r = size_\delta \in \mathbb{Q}\langle\langle T_\Sigma \rangle\rangle$ be defined as in the lecture. Choose an appropriate basis of V_r and use the constructive proof from the lecture to obtain an automaton $\mathcal{A}$ with $r_\mathcal{A} = r$. $\diamond$