Second exercise sheet for the lecture

Syntax Directed Semantics (Part II)

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Exercise 4 (multilinear mappings)
Consider the $\mathbb{Q}$-vector space $V = \mathbb{Q}^3$. Give some multilinear mappings of type $V^3 \to V$. $\diamond$

Exercise 5 (linear algebra)
Let S be a field. Let V_1 and V_2 be S-vector spaces.

1. Show that if V_1 is a subspace of V_2, then $\dim(V_1) \leq \dim(V_2)$.
2. Let $\phi : V_1 \to V_2$ be a linear transformation. Show that $\dim(\mathrm{im}(\phi)) \leq \dim(V_1)$. $\diamond$

Exercise 6 (left-quotients)
Let $\Sigma = \{\sigma^{(2)}, \delta^{(1)}, \alpha^{(0)}\}$ and let $r = \mathit{size}_\delta \in \mathbb{Q}\langle\langle T_\Sigma\rangle\rangle$ be defined as in the lecture.

1. Let $\xi = \sigma(\delta(\sigma(\alpha, \alpha)), \alpha)$. Characterize $\xi^{-1}r \in \mathbb{Q}\langle\langle C_\Sigma^*\rangle\rangle$.
2. Characterize all elements of V_r.
3. Determine a finite basis of V_r.

Try to perform the same tasks for $r = \mathit{height} \in \mathbb{Q}\langle\langle T_\Sigma\rangle\rangle$. $\diamond$

Exercise 7 (claim from lecture)
Let S be a field, Σ be a ranked alphabet, and $r \in S\langle\langle T_\Sigma \rangle\rangle$. Let $k, n \geq 0$, $\sigma \in \Sigma^{(k)}$, $\xi_1, \ldots, \xi_k, \eta_1, \ldots, \eta_n \in T_\Sigma$, and $a_{1,1}, \ldots, a_{k,n} \in S$ be such that for every $1 \leq i \leq k$,

$$\xi_i^{-1} r = a_{i,1}(\eta_1^{-1} r) + \cdots + a_{i,n}(\eta_n^{-1} r)$$

Show that then for every $1 \leq l \leq k$ and $1 \leq j_1, \ldots, j_{l-1} \leq n$,

$$\sum_{1 \leq j_l \leq n} a_{l,j_l}(\sigma(\eta_{j_1}, \ldots, \eta_{j_{l-1}}, \eta_{j_l}, \xi_{l+1}, \ldots, \xi_k)^{-1} r)$$
$$=$$
$$\sigma(\eta_{j_1}, \ldots, \eta_{j_{l-1}}, \xi_l, \xi_{l+1}, \ldots, \xi_k)^{-1} r \qquad \diamond$$