Tenth exercise sheet for the lecture

Syntax Directed Semantics (Part II)

Prof. Dr. Heiko Vogler / Dr. Janis Voigtländer

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Exercise 29
Let $\underline{K} = (K, \oplus, \odot, 0, 1)$ be a finite, commutative semiring and let $\Sigma = \{\oplus^{(2)}, \odot^{(2)}\} \cup \{k^{(0)} \mid k \in K\}$. Try to give a closed formula φ such that for every $t \in T_\Sigma$, $([\![\varphi]\!], t)$ is the "evaluation" of t in $\underline{K}$. $\diamond$

Exercise 30
Show that the class of recognizable step functions is closed under both $+$ and $\odot$. $\diamond$

Exercise 31
Let $\Sigma = \{\gamma^{(1)}, \alpha^{(0)}\}$, consider the semiring $\mathbb{N}$, and let $\varphi = \forall x.\forall y.2$.

1. Show that φ is not restricted.
2. Show that $[\![\varphi]\!]$ is not recognizable. $\diamond$

Exercise 32
Let $\varphi \in \mathit{MSO}(K, \Sigma)$ with $\mathit{Free}(\varphi) = \mathcal{W}$, and let $M = (Q, \Sigma_\mathcal{W}, K, \mu, \gamma)$ be a wta with $r_M = [\![\varphi]\!]$. Construct a wta M' with $r_{M'} = [\![\exists X.\varphi]\!]$. $\diamond$

Exercise 33
Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$, $\mathcal{W} = \{x\}$, and let the languages $L_1, L_2, L_3 \subseteq T_{\Sigma_\mathcal{W}}$ be given as follows:

- $L_1 = L(\varphi)$, where $\varphi = \mathit{label}_\gamma(x)$.
- $L_2 = L(M)$, where $M = (\{q, q'\}, \Sigma_\mathcal{W}, \delta, \{q\})$ with $\delta^0_\alpha() = \delta^0_{\langle\alpha,x\rangle}() = \{q\}$, $\delta^1_\gamma(q) = \delta^1_\gamma(q') = \delta^1_{\langle\gamma,x\rangle}(q) = \delta^1_{\langle\gamma,x\rangle}(q') = \{q'\}$, $\delta^2_\sigma(q,q) = \delta^2_{\langle\sigma,x\rangle}(q,q) = \{q\}$, and $\delta^2_\sigma(-,-) = \delta^2_{\langle\sigma,x\rangle}(-,-) = \{q'\}$ for every other combination.
- $L_3 = T_{\Sigma_\mathcal{W}} \setminus (L_1 \cup L_2)$.

Construct an fta $\widetilde{M}$ such that

$$L(\widetilde{M}) = \{(s', \nu) \in T_{\Sigma \times \{1,2,3\}} \mid \forall w \in pos(s').\ s'[x \to w] \in L_{\nu(w)}\}. \qquad \diamond$$

Exercise 34

Let Σ be a ranked alphabet, $\mathcal{V}$ be a finite set of variables, and $\varphi, \psi \in$ qf-$MSO^{-}(\Sigma)$ with $\mathit{Free}(\varphi), \mathit{Free}(\psi) \subseteq \mathcal{V}$.

1. Show that $[\![\varphi^{+} \wedge \varphi^{-}]\!]_{\mathcal{V}} = [\![0]\!]_{\mathcal{V}}$ and $[\![\varphi^{+} \vee \varphi^{-}]\!]_{\mathcal{V}} = [\![1]\!]_{\mathcal{V}}$.
2. Give similar simplifications for $[\![\varphi \wedge \varphi^{-}]\!]_{\mathcal{V}}$ and $[\![\varphi \wedge \varphi^{+}]\!]_{\mathcal{V}}$.
3. Show that $[\![\varphi \to \varphi]\!]_{\mathcal{V}} = [\![\varphi \vee \varphi^{-}]\!]_{\mathcal{V}}$.
4. Show that $[\![\varphi \leftrightarrow \psi]\!]_{\mathcal{V}} = [\![(\varphi \wedge \psi) \vee (\varphi^{-} \wedge \psi^{-})]\!]_{\mathcal{V}}$. $\diamond$