First exercise sheet for the lecture

Syntax Directed Semantics (Part II)

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Exercise 1 (wtas)

1. Let $\underline{S} = (S, \oplus, \odot, 0, 1)$ be a finite semiring. Consider the wta $\mathcal{A} = (Q, \Sigma, \underline{S}, \mu, \nu)$, where $Q = \{q_1, q_2\}$, $\Sigma = \{\oplus^{(2)}, \odot^{(2)}\} \cup \{a^{(0)} \mid a \in S\}$, $\nu(q_1) = 1$, $\nu(q_2) = 0$, for every $a \in S$, $\mu_0(a)_{\varepsilon,q_1} = a$ and $\mu_0(a)_{\varepsilon,q_2} = 1$, and μ_2 is given as follows:

$\mu_2(\oplus)$	q_1	q_2
q_1q_1	0	0
q_1q_2	1	0
q_2q_1	1	0
q_2q_2	0	1

$\mu_2(\odot)$	q_1	q_2
q_1q_1	1	0
q_1q_2	0	0
q_2q_1	0	0
q_2q_2	0	1

   Describe the induced formal tree series $S_{\mathcal{A}}$.

2. For general Σ and arbitrary $\delta \in \Sigma$, give a wta over $\mathbb{Q}$ that recognizes $size_\delta$.

3. For $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$, give a wta over a suitable semiring that recognizes *height*. ◇

Exercise 2 (linear transformations vs. multilinear mappings)

Prove or disprove (using concrete counterexamples):

1. Every linear transformation of the form $f : V^m \to V$ is a multilinear mapping.

2. Every multilinear mapping is a linear transformation. ◇

Exercise 3 (characterization of linear forms)

Let V be a vector space over S with finite basis B. Identify every $v \in V \cong S^B$ with the associated "column vector". Show that every linear form $\lambda : V \to S$ is precisely characterized by multiplication with a fixed "row vector". ◇