Eighth exercise sheet for the lecture

# The Algebraic Path Problem

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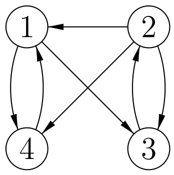
**Exercise 25 (infinitary sum operations and complete semirings)**

5. Is there a semiring $K$ such that there are two different infinitary sum operations $\sum^1$ and $\sum^2$ with both $(K, \sum^1)$ and $(K, \sum^2)$ being complete?

6. Is there a (finite) semiring such that there is no infinitary sum operation making it complete? $\diamond$

**Exercise 27 (Matrix iteration)**
Consider the following graph:




Let it be weighted over the complete semiring $(\mathbb{N}_\infty, max, \cdot, 0, 1)$, with supremum as infinitary sum operation, as follows:

| $e$ | $(1,3)$ | $(1,4)$ | $(2,1)$ | $(2,3)$ | $(2,4)$ | $(3,2)$ | $(4,1)$ |
|---|---|---|---|---|---|---|---|
| $\lambda(e)$ | 0 | 1 | 3 | 1 | 4 | 2 | 1 |

Compute $(M_G)^*$. $\diamond$

**Exercise 28 (the star operation)**
Describe $a^*$ for an element $a$ from any of the following complete semirings (with the "canonical" infinitary sum operations):

$(\{0,1\}, \vee, \wedge, 0, 1)$
$([0,1], max, \cdot, 0, 1)$
$(\mathbb{N}_\infty, +, \cdot, 0, 1)$
$(\mathbb{N}_\infty, max, \cdot, 0, 1)$
$(\mathbb{R}_\infty^{\geq 0}, max, min, 0, \infty)$
$(\mathcal{P}(\Sigma^*), \cup, \cap, \emptyset, \Sigma^*)$
$(\mathcal{P}(\Sigma^*), \cup, \cdot, \emptyset, \{\varepsilon\})$ where $\cdot$ is language concatenation ◇

**Exercise 29 (Gauss-Jordan algorithm)**
Execute the Gauss-Jordan algorithm for the weighted graph from Exercise 27. ◇