Sixth exercise sheet for the lecture

The Algebraic Path Problem

Prof. Dr. Heiko Vogler / Dr. Janis Voigtländer

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Exercise 21 (the smallest tree problem)
Consider the superior Σ-hypergraph $H = (V, E, F)$ with $\Sigma = \{\alpha^{(0)}, \beta^{(0)}, \gamma^{(1)}, \kappa^{(1)}, \sigma^{(2)}\}$, $V = \{1, 2, 3\}$, $E = \{(\varepsilon, \alpha, 1), (\varepsilon, \beta, 2), (1, \gamma, 2), (1, \kappa, 3), (12, \sigma, 3), (2, \gamma, 3), (3, \gamma, 2)\}$, and $F = \{f_\alpha, f_\beta, f_\gamma, f_\kappa, f_\sigma\}$ as follows:

$$\begin{array}{lcl@{\qquad\qquad}lcl} f_\alpha() & = & 2 & f_\kappa(x_1) & = & 2 \cdot x_1 + 3 \\ f_\beta() & = & 4 & f_\sigma(x_1, x_2) & = & x_1 + x_2 \\ f_\gamma(x_1) & = & 1 + x_1 & & & \end{array}$$

Compute the smallest tree values $m(1)$, $m(2)$, and $m(3)$. ◇

Exercise 22 (“hypercycles”)
Define a notion of cycles in Σ-hypergraphs such that in the absence of such cyles T_H is guaranteed to be finite. ◇

Exercise 23 (Knuth algorithm)

1. Execute the Knuth algorithm on the superior Σ-hypergraph from Exercise 21.
2. Sketch how the Knuth algorithm can be adapted to additionally provide actual smallest trees.
3. Argue that the Knuth algorithm relaxes every hyperedge at most once. Is it true that the Knuth algorithm relaxes every non-nullary hyperedge exactly once if the hypergraph is (somehow) acyclic?
4. Give an example showing that the Knuth algorithm may fail to produce correct results if not all edge-weighting functions are superior.
5. Give an example showing that the Knuth algorithm may fail to produce correct results if not all edge-weighting functions are superior, but each of them is maximizing. ◇