Fifth exercise sheet for the lecture

The Algebraic Path Problem

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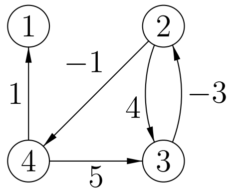
Exercise 15 (Iteration algorithm)

1. Execute the Iteration algorithm with doubling up on the following $\mathbb{R}$-weighted graph:



2. Give an example ($\mathbb{R}$-weighted graph) so that one of the matrices encountered in the Iteration algorithm would have a negative value on its diagonal.

3. Sketch how the Iteration algorithm (with or without doubling up) can be adapted to additionally provide actual shortest paths. ◇

Exercise 16 (Floyd-Warshall algorithm)
Execute the Floyd-Warshall algorithm on the $\mathbb{R}$-weighted graph from Exercise 15(1). ◇

Exercise 17 (variations of the Floyd-Warshall algorithm)

1. Discuss whether the Floyd-Warshall algorithm would still be correct if performing the assignment to $\delta_{u,v}^{(l)}$ only for $u \neq v$.

2. Discuss whether the Floyd-Warshall algorithm would still be correct if performing the outermost loop one time less.

3. Can you conceive a "doubling up" variant of the Floyd-Warshall algorithm? ◇

Exercise 18 (comparison of algorithms)
To which of the following statements would you agree when considering the all-pairs shortest path problem?

1. For graphs without negative weights, the Dijkstra algorithm (repeated for every source vertex) is the best choice.
2. The Bellman-Ford algorithm (repeated for every source vertex) performs too bad on dense graphs. Hence, it is not worthwhile in practice.
3. The Iteration algorithm (even with doubling up) is generally worse than repeated application of the Bellman-Ford algorithm.
4. The Floyd-Warshall algorithm is the best choice if we know there are no negative weighted cycles. ◇

Exercise 19 (hypergraphs and trees leading to vertices)
Let $\Sigma = \{\alpha^{(0)}, \beta^{(0)}, \gamma^{(1)}, \sigma^{(2)}\}$, $V = \{1, 2, 3\}$, and $E = \{(\varepsilon, \alpha, 1), (\varepsilon, \beta, 2), (2, \gamma, 1), (3, \gamma, 1), (21, \sigma, 3)\}$.

1. Draw the Σ-hypergraph $H = (V, E)$.
2. Give the sets T_H^1, T_H^2, and T_H^3. ◇

Exercise 20 (superior functions)
Let $c \in \mathbb{R}_\infty^{\geq 0}$. Which of the following functions are superior?

$$
\begin{aligned}
f_1() &= c \\
f_2(x_1) &= c \\
f_3(x_1) &= x_1 + c \\
f_4(x_1) &= c \cdot x_1 \\
f_5(x_1) &= x_1^c \\
f_6(x_1, x_2) &= c \cdot x_1 \\
f_7(x_1, x_2) &= x_1 \cdot x_2 \\
f_8(x_1, x_2) &= max(x_1, x_2) \\
f_9(x_1, x_2) &= max(1, x_1) \cdot max(1, x_2) \\
f_{10}(x_1, x_2) &= x_1 + x_2 + c \\
f_{11}(x_1, x_2) &= x_1 + x_2 - min\{x_1, c \cdot x_2\}
\end{aligned}
$$
◇