Fourth exercise sheet for the lecture

The Algebraic Path Problem

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Exercise 10 (variations of the Dijkstra algorithm)

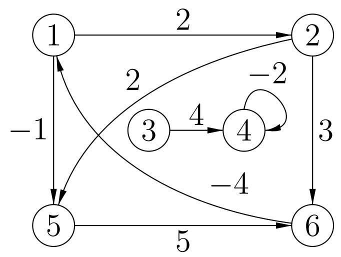
4. Discuss whether the Dijkstra algorithm would still be correct if the loop condition "`while` $U \neq \emptyset$" would be changed into "`while` $|U| > 1$" or "`while` $|U| > 2$".

5. Discuss the behavior of the Dijkstra algorithm if ∞ is additionally allowed as edge weight in the input graph.

6. Argue that the "general/algebraic" variant of the Dijkstra algorithm considered last week is only applicable if the partial order occurring in the problem domain is actually a total order. ◇

Exercise 11 (Bellman-Ford algorithm)

Give a protocol of executing the Bellman-Ford algorithm (adapted to additionally provide actual shortest paths) on the following $\mathbb{R}$-weighted graph with source vertex 1:



Use the following linear order for relaxing edges in each pass:

$$(1,2); (1,5); (2,5); (2,6); (3,4); (4,4); (5,6); (6,1).$$ ◇

Exercise 12 (one direction of Lemma 3.9)
Let $G = (V, E, \lambda)$ be an $\mathbb{R}$-weighted graph and $s \in V$ the fixed source vertex. Prove that for every vertex $v \in V$, if there is a path from s to v, then $\delta(v) < \infty$ after termination of the Bellman-Ford algorithm. $\diamond$

Exercise 13 (variations of the Bellman-Ford algorithm)

1. Discuss the behavior of the Bellman-Ford algorithm if ∞ is additionally allowed as edge weight in the input graph.

2. Discuss whether the Bellman-Ford algorithm would still be correct if the relaxation on all edges would be performed only $|V| - 2$ times.

3. Discuss whether the Bellman-Ford algorithm would still be correct if performing *relax*(u, v) only for $(u, v) \in E$ with $u \neq v$.

4. Discuss whether the Bellman-Ford algorithm would still be correct if performing the check $\delta(v) > \delta(u) + \lambda(u, v)$ only for $(u, v) \in E$ with $u \neq v$. $\diamond$

Exercise 14 (properties of the Bellman-Ford algorithm)

1. Argue that the Bellman-Ford algorithm can be aborted if in some iteration over all edges there was no change to δ. What will be the outcome of the algorithm in this case?

2. Discuss what information can be retrieved from the δ-function if the Bellman-Ford algorithm terminates with "negative weighted cycle".

3. Give an example showing that not every vertex taking part in a negative weighted cycle is necessarily detected by the test $\delta(v) > \delta(u) + \lambda(u, v)$ in line 5 of the Bellman-Ford algorithm. $\diamond$