Third exercise sheet for the lecture

The Algebraic Path Problem

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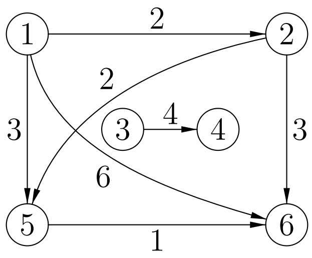
Exercise 8 (Dijkstra algorithm)
Consider the following $\mathbb{R}^{\geq 0}$-weighted graph:



Give a protocol of executing the Dijkstra algorithm with source vertex 1. $\diamond$

Exercise 9 (properties of the Dijkstra algorithm)

1. Give a sufficient criterion on graphs such that the Dijkstra algorithm does not relax every edge.

2. Give a sufficient criterion on graphs and source vertices such that the Dijkstra algorithm does relax every edge.

3. Give an example showing that in the Dijkstra algorithm (contrary to the Bellman-Ford algorithm) the sequence/multiset of performed relaxations depends on the edge weights, not just on the underlying unweighted graph (and the source vertex).

4. Give an example showing that the Dijkstra algorithm may fail to produce correct results if negative weighted edges are present.

5. Argue that the Dijkstra algorithm never relaxes an edge (u, u). $\diamond$

Exercise 10 (variations of the Dijkstra algorithm)

1. Adapt the Dijkstra algorithm in order to provide, for every vertex in the graph, an actual shortest path to it from the source vertex (rather than just providing the weight of such a shortest path).

   Give a protocol of executing the adapted algorithm on the graph from Exercise 8 with source vertex 1.

2. Adapt the Dijkstra algorithm in order to compute, for $[0,1]$-weighted graphs, the "highest attainable information transfer reliability" between a given source vertex and every other vertex.

3. Try to give general criteria (on the operations involved in a problem domain) guaranteeing that an adaptation of the Dijkstra algorithm as in the previous point is possible.

4. Discuss whether the Dijkstra algorithm would still be correct if the loop condition "`while` $U \neq \emptyset$" would be changed into "`while` $|U| > 1$" or "`while` $|U| > 2$". $\diamond$