Second exercise sheet for the lecture

The Algebraic Path Problem

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Exercise 4 (the single-source shortest path problem)

1. Consider the following $\mathbb{R}^{\geq 0}$-weighted graph:

$$
\begin{tikzcd}
1 \arrow[rr, "2"] \arrow[dd, "3"'] \arrow[ddrr, bend right, "6"] & & 2 \arrow[dd, "3"] \arrow[ddll, bend right, "2"'] \\
& 3 \arrow[r, "4"] & 4 \\
5 \arrow[rr, "1"'] & & 6
\end{tikzcd}
$$

   Give the shortest-path weight $d_{1,t}$ for every $t \in \{1, 2, 3, 4, 5, 6\}$.

2. Based on the above graph, give an example for a vertex s and an edge (t, u) with $d_{s,u} \neq d_{s,t} + \lambda(t, u)$. ◇

Exercise 5 (relaxations)

Given the graph from Exercise 4 and the function $\delta : \{1, 2, 3, 4, 5, 6\} \to \mathbb{R}^{\geq 0}_{\infty}$ as follows:

v	1	2	3	4	5	6
$\delta(v)$	0	2	∞	∞	3	6

perform the following sequence of relaxations:

$$\mathit{relax}(2, 5); \mathit{relax}(2, 6); \mathit{relax}(5, 6); \mathit{relax}(3, 4).$$ ◇

Exercise 6 (auxiliary lemmas)

1. Show that the condition $u \neq v$ is necessary in Lemma 3.3.

2. Show that it is essential that δ^{in} is really the function from the lecture (not an arbitrary function of type $V \to \mathbb{R}_\infty$) in Lemmas 3.4 to 3.6.

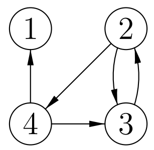
3. Show that the statement of Lemma 3.6 does not in general hold if $s \Rightarrow u \to v$ is any path from s to v (not necessarily a shortest one).

4. Discuss the case $u = v$ for Lemma 3.6. ◇

Exercise 7 (reflexive, transitive closure)

Let $G = (V, E)$ be a graph. The *reflexive, transitive closure* of G is the graph $G^* = (V, E')$ with $E' = \{(u, v) \in V \times V \mid P_{u,v} \neq \emptyset\}$, where $P_{u,v}$ refers to G-paths.

1. Give the reflexive, transitive closure of the following graph:



2. How can the reflexive, transitive closure of a graph be computed using a variation of matrix multiplication? ◇