Twelfth exercise sheet for the lecture

The Algebraic Path Problem

Prof. Dr. Heiko Vogler / Dr. Janis Voigtländer

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Exercise 41 (local finiteness and φ-Cauchy-product)
Let Δ be an alphabet, $(K, +, \cdot, 0, 1)$ be a semiring, and $\varphi : K \to K$ be an endomorphism on it. Further, let $(S_n \mid n \geq 1)$ be a locally finite family of power series over Δ^+ and K, and $k \in K$. Show that then:

$$\Big(\sum_{n\geq 1} S_n\Big) \odot_\varphi k.\varepsilon = \sum_{n\geq 1}(S_n \odot_\varphi k.\varepsilon).$$

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Exercise 42 (φ-rational formal power series)
Let $\Delta = \{\alpha, \beta, \gamma\}$, $K = (\mathbb{N}_\infty, max, \cdot, 0, 1)$, and $\varphi(x) = x^2$. Describe the following formal power series $S \in K\langle\langle\Delta^*\rangle\rangle$:

$$(S, w) = \begin{cases} 17 & \text{if } w \in \{\varepsilon, \alpha\beta\gamma\} \\ 2^{2^{n+1}} & \text{if } w = \alpha^n\beta\gamma \text{ for some } n \in \{2, 3\} \\ 0 & \text{otherwise} \end{cases}$$

by a (preferrably compact) expression over the semiring $K_\varphi\langle\langle\Delta^*\rangle\rangle$ of skew formal power series.

Try the same for the following formal power series:

$$(S, w) = \begin{cases} 17 & \text{if } w \in \{\varepsilon, \alpha\beta\gamma\} \\ 2^{2^{n+1}} & \text{if } w = \alpha^n\beta\gamma \text{ for some } n \geq 2 \\ 0 & \text{otherwise.} \end{cases}$$

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Exercise 43 (closure of $\boldsymbol{K}^{\mathbf{rec},\varphi}\langle\langle\boldsymbol{\Delta}^*\rangle\rangle$ under $+$ and $\odot_\varphi$)
Let Δ be an alphabet, $(K, +, \cdot, 0, 1)$ be a semiring, $\varphi : K \to K$ be a semiring endomorphism, and M_1 and M_2 be weighted automata over Δ and K. Show that there are a weighted automata N and N' over Δ and K with:

1. $S^\varphi_N = S^\varphi_{M_1} + S^\varphi_{M_2}$.
2. $S^\varphi_{N'} = S^\varphi_{M_1} \odot_\varphi S^\varphi_{M_2}$.

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