Eleventh exercise sheet for the lecture

The Algebraic Path Problem

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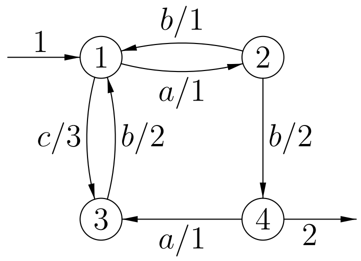
Exercise 37 (semiring endomorphisms)

1. What semiring endomorphisms are there on $\underline{\mathbb{B}} = (\{0,1\}, \vee, \wedge, 0, 1)$?

2. Discuss some potential endomorphisms on the following semirings:

$([0,1], max, \cdot, 0, 1)$
$(\mathbb{N}_\infty, max, \cdot, 0, 1)$
$(\mathbb{N}_\infty, +, \cdot, 0, 1)$
$(\mathbb{R}^{\geq 0}_{\infty}, max, min, 0, \infty)$
$(\mathbb{R}^{\geq 0}_{-\infty}, max, +, -\infty, 0)$
$(\mathcal{P}(\Sigma^*), \cup, \cap, \emptyset, \Sigma^*)$
$(\mathcal{P}(\Sigma^*), \cup, \cdot, \emptyset, \{\varepsilon\})$ where $\cdot$ is language concatenation. ◇

Exercise 38 (run semantics)
Consider the following weighted automaton M over $(\mathbb{N}_\infty, max, \cdot, 0, 1)$:



Compute $(S^\varphi_M, abcbab)$ and $(S^\varphi_M, ababab)$, where $\varphi(x) = x^2$. ◇

Exercise 39 (φ-recognizable formal power series)
Let $\Delta = \{\alpha, \beta, \gamma\}$, $K = (\mathbb{N}_\infty, max, \cdot, 0, 1)$, and $\varphi(x) = x^2$. Show that the following formal power series $S \in K\langle\langle\Delta^*\rangle\rangle$ is φ-recognizable:

$$(S, w) = \begin{cases} 17 & \text{if } w \in \{\varepsilon, \alpha\beta\gamma\} \\ 2^{2^{n+1}} & \text{if } w = \alpha^n\beta\gamma \text{ for some } n \geq 2 \\ 0 & \text{otherwise} \end{cases}$$ ◇

Exercise 40 (φ-recognizable formal power series)
Let Δ be an alphabet. A formal power series $S \in \mathbb{R}^{\geq 0}_{-\infty}\langle\langle \Delta^* \rangle\rangle$ is said to be *bounded from above* if there is a $k \in \mathbb{R}^{\geq 0}_{-\infty}$ such that $(S, w) \leq k$ for every $w \in \Delta^*$. Now, let $d \in [0, 1)$ and let φ be the semiring endomorphism on $\underline{Arct}_{\mathbb{R}^{\geq 0}} = (\mathbb{R}^{\geq 0}_{-\infty}, max, +, -\infty, 0)$ defined by $\varphi(x) = d \cdot x$. Show that if a formal power series is φ-accepted by a weighted automaton over $\underline{Arct}_{\mathbb{R}^{\geq 0}}$, then it is bounded from above. $\diamond$